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Type Package

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Description Implements algorithms for analyzing Cayley graphs of permutation groups for the TopSpin puzzle. Provides methods for cycle detection, state space exploration, bidirectional BFS pathfinding, and finding optimal operation sequences in permutation groups generated by shift and reverse operations. Includes C++ implementations of core operations via 'Rcpp' for performance, including a depth-limited BFS path shortener implemented entirely in C++.

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analyze_top_combinations
Analyze Top Operation Combinations

Description

For each combination in a data frame of top results, runs a full cycle analysis and collects all states with their celestial coordinates into a single data frame.

Usage

```
analyze_top_combinations(top_combos, start_state, k)
```

Arguments

top_combos	Data frame or data.table with a combination column (string of operation digits, e.g., "132")
start_state	Integer vector, the initial permutation state
k	Integer, parameter for reverse operations

Value

Data frame with columns V1..Vn, operation, step, combo_number, nL, nR, nX, theta, phi, omega_conformal

Examples

```
combos <- data.frame(combination = c("13", "23"), stringsAsFactors = FALSE)
# result <- analyze_top_combinations(combos, 1:10, k = 4)
```

apply_operations	<i>Apply Sequence of Operations</i>
------------------	-------------------------------------

Description

Applies a sequence of shift and reverse operations to a permutation state. Operations can be specified as "1"/"L" (shift left), "2"/"R" (shift right), or "3"/"X" (reverse prefix). Tracks celestial coordinates.

Arguments

state	Integer vector representing the current permutation state
operations	Character vector of operations ("1"/"L", "2"/"R", "3"/"X")
k	Integer, parameter for reverse operations
coords	Optional list of current celestial coordinates. If NULL, starts from zero coordinates.

Value

List with components:

state	Integer vector after all operations applied
coords	List of final celestial coordinates (nL, nR, nX, theta, phi, omega_conformal)

Examples

```
result <- apply_operations(1:10, c("1", "3", "2"), k = 4)
result$state

# Using letter codes
result <- apply_operations(1:20, c("L", "X", "R"), k = 4)
result$state
```

apply_operations_batch_gpu	<i>Apply operations to batch of states on GPU</i>
----------------------------	---

Description

Applies a sequence of permutation operations to multiple states simultaneously using matrix multiplication on the Vulkan backend.

Usage

```
apply_operations_batch_gpu(states_matrix, operations, k)
```

Arguments

states_matrix Numeric matrix (nrow x n), each row is a state
 operations Character vector of operation codes (e.g., c("L", "R", "X"))
 k Integer, parameter for reverse operations

Value

Numeric matrix (nrow x n) with transformed states

Examples

```
if (cayley_gpu_available()) {
  mat <- matrix(c(1,2,3,4,5, 5,4,3,2,1), nrow = 2, byrow = TRUE)
  result <- apply_operations_batch_gpu(mat, c("1", "3"), k = 4)
}
```

bidirectional_bfs *Bidirectional BFS Shortest Path*

Description

Finds the shortest path between two permutation states using bidirectional breadth-first search. Expands from both the start and goal states simultaneously, meeting in the middle.

Usage

```
bidirectional_bfs(n, state1, state2, max_level, moves, k)
```

Arguments

n Integer, size of the permutation
 state1 Integer vector, start state
 state2 Integer vector, goal state
 max_level Integer, maximum BFS depth in each direction
 moves Character vector, allowed operations (e.g., c("1", "2", "3"))
 k Integer, parameter for reverse operations

Value

Character vector of operations forming the shortest path, or NULL if no path found within max_level

Examples

```
# Find path between two small states
path <- bidirectional_bfs(5, 1:5, c(2, 3, 4, 5, 1), max_level = 5,
  moves = c("1", "2", "3"), k = 3)
path
```

breakpoint_distance *Breakpoint Distance Between Two States*

Description

Counts the number of positions where consecutive elements differ by more than 1 (breakpoints). Particularly effective for TopSpin puzzles where operations shift blocks and flip prefixes.

Usage

```
breakpoint_distance(start_state, target_state)
```

Arguments

start_state Integer vector, first state
target_state Integer vector, second state

Value

Integer, the number of breakpoints

Examples

```
breakpoint_distance(1:5, 5:1)
breakpoint_distance(1:5, 1:5)
```

calculate_angular_distance_z
Angular Distance Between Two Celestial Points

Description

Computes the angular distance on the celestial sphere between two points given as coordinate lists (each with a z component).

Usage

```
calculate_angular_distance_z(result1, result2)
```

Arguments

result1 List with component z (complex number)
result2 List with component z (complex number)

Value

Numeric, angular distance in radians

Examples

```
c1 <- convert_LRX_to_celestial(10, 5, 3)
c2 <- convert_LRX_to_celestial(1, 1, 2)
calculate_angular_distance_z(c1, c2)
```

calculate_differences *Calculate Manhattan Distances for All States*

Description

Computes the Manhattan distance from a reference state to every row in a table of reachable states, adds a difference column, and sorts by it.

Usage

```
calculate_differences(  
  start_state,  
  reachable_states_start,  
  method = "manhattan",  
  use_gpu = FALSE  
)
```

Arguments

start_state	Integer vector, the reference state
reachable_states_start	Data frame with V-columns
method	Character, distance method (currently only "manhattan")
use_gpu	Logical, use GPU acceleration via ggmlR if available (default FALSE)

Value

Data frame sorted by difference (ascending)

Examples

```
df <- data.frame(V1 = c(1, 2), V2 = c(2, 1))
calculate_differences(c(1, 2), df)
```

calculate_midpoint_z *Midpoint Between Two Celestial Coordinates*

Description

Computes the midpoint on the celestial sphere between two coordinate sets by averaging Cartesian unit-sphere positions and re-projecting.

Usage

```
calculate_midpoint_z(coords1, coords2)
```

Arguments

coords1 List with theta, phi, omega_conformal (and optionally nL, nR, nX)
coords2 List with theta, phi, omega_conformal (and optionally nL, nR, nX)

Value

List with theta, phi, z, z_bar, omega_conformal, nL, nR, nX

Examples

```
c1 <- convert_LRX_to_celestial(10, 5, 3)  
c2 <- convert_LRX_to_celestial(1, 1, 2)  
mid <- calculate_midpoint_z(c1, c2)  
mid$theta
```

cayley_gpu_available *Check if GPU acceleration is available*

Description

Checks whether ggmR is installed and Vulkan GPU is present.

Usage

```
cayley_gpu_available()
```

Value

Logical

Examples

```
cayley_gpu_available()
```

cayley_gpu_free	<i>Free GPU backend resources</i>
-----------------	-----------------------------------

Description

Free GPU backend resources

Usage

```
cayley_gpu_free()
```

Value

Invisible NULL

cayley_gpu_init	<i>Initialize GPU backend</i>
-----------------	-------------------------------

Description

Lazily initializes the Vulkan backend. Safe to call multiple times.

Usage

```
cayley_gpu_init(device = 0L, force = FALSE)
```

Arguments

device	Integer, Vulkan device index (0-based)
force	Logical, force re-initialization

Value

Invisible backend pointer

Examples

```
if (cayley_gpu_available()) {  
  cayley_gpu_init()  
}
```

cayley_gpu_status	<i>Get GPU status information</i>
-------------------	-----------------------------------

Description

Get GPU status information

Usage

```
cayley_gpu_status()
```

Value

List with availability, device info, and backend status

Examples

```
cayley_gpu_status()
```

check_duplicates	<i>Find Duplicate States Between Two Tables</i>
------------------	---

Description

Identifies states that appear in both tables by comparing V-columns. Used for finding intersections between forward and backward searches.

Usage

```
check_duplicates(df1, df2)
```

Arguments

df1	Data frame (first set of states)
df2	Data frame (second set of states)

Value

Data frame of duplicate states with a source column, or NULL if none

Examples

```
df1 <- data.frame(V1 = c(1, 2), V2 = c(2, 1))  
df2 <- data.frame(V1 = c(2, 3), V2 = c(1, 2))  
check_duplicates(df1, df2)
```

convert_digits	<i>Convert String to Integer Vector of Digits</i>
----------------	---

Description

Parses a string of digits or space-separated numbers into an integer vector. Useful for converting operation sequences or state representations.

Usage

```
convert_digits(s)
```

Arguments

s	Character string. Either a string of single digits (e.g., "123") or space-separated numbers (e.g., "1 2 3" or "10 11 12").
---	--

Value

Integer vector of parsed numbers

Examples

```
convert_digits("123")
convert_digits("1 5 4 3 2")
convert_digits("10 11 12 13")
```

convert_LRX_to_celestial	<i>Convert LRX Counts to Celestial Coordinates</i>
--------------------------	--

Description

Maps cumulative operation counts (Left, Right, Reverse) to spherical celestial coordinates via stereographic projection.

Usage

```
convert_LRX_to_celestial(nL, nR, nX)
```

Arguments

nL	Integer, cumulative count of left shift operations
nR	Integer, cumulative count of right shift operations
nX	Integer, cumulative count of reverse operations

Value

List with components:

z	Complex number, stereographic projection coordinate
z_bar	Complex conjugate of z
theta	Numeric, zenith angle (from X axis)
phi	Numeric, azimuthal angle (in LR plane)
omega_conformal	Numeric, conformal energy (magnitude of momentum vector)

Examples

```
coords <- convert_LRX_to_celestial(10, 5, 3)
coords$theta
coords$phi
```

create_state_store *Create a New State Store*

Description

Creates a C++ StateStore object for compact, incremental storage of permutation states with hash-indexed lookup.

Usage

```
create_state_store(perm_length, init_capacity = 10000L)
```

Arguments

perm_length	Integer, length of each permutation state
init_capacity	Integer, initial capacity (default 10000)

Value

External pointer to StateStore (XPtr)

Examples

```
store <- create_state_store(6L)
state_store_size(store)
```

 find_best_random_combinations

Find Best Random Operation Sequences

Description

Generates random sequences of operations and evaluates their cycle lengths to find sequences that produce the best cycles in the Cayley graph. Uses C++ with OpenMP for parallel evaluation of combinations.

Usage

```
find_best_random_combinations(
    moves,
    combo_length,
    n_samples,
    n_top,
    start_state,
    k,
    sort_by = c("longest", "most_unique")
)
```

Arguments

moves	Character vector of allowed operation symbols (e.g., c("1", "2", "3") or c("L", "R", "X"))
combo_length	Integer, length of each operation sequence to test
n_samples	Integer, number of random sequences to generate and test
n_top	Integer, number of top results to return
start_state	Integer vector, initial permutation state
k	Integer, parameter for reverse operations
sort_by	Character vector of sorting criteria, applied in order. Available criteria: "longest" Prefer longer cycles (descending total_moves) "shortest" Prefer shorter cycles (ascending total_moves) "most_unique" Prefer more unique states (descending unique_states_count) "least_unique" Prefer fewer unique states (ascending unique_states_count) "most_repeated" Prefer higher repetition ratio total_moves/unique_states_count (descending) "least_repeated" Prefer lower repetition ratio (ascending) Default: c("longest", "most_unique") (original behavior).

Value

Data frame with columns:

combo_number	Integer sequence number
combination	String representation of the operation sequence
total_moves	Cycle length for this sequence
unique_states_count	Number of unique states visited in the cycle
repetition_ratio	Ratio total_moves / unique_states_count

Examples

```
# Default: longest cycles
best <- find_best_random_combinations(
  moves = c("1", "2", "3"),
  combo_length = 10,
  n_samples = 50,
  n_top = 5,
  start_state = 1:10,
  k = 4
)

# Short cycles with many unique states
best2 <- find_best_random_combinations(
  moves = c("1", "2", "3"),
  combo_length = 10,
  n_samples = 50,
  n_top = 5,
  start_state = 1:10,
  k = 4,
  sort_by = c("shortest", "most_unique")
)
```

find_closest_to_coords

Find Closest State to Target Coordinates

Description

Searches a table of reachable states for the state whose celestial coordinates are closest to a target coordinate set.

Usage

```
find_closest_to_coords(reachable_states, target_coords, v_cols)
```

Arguments

reachable_states Data frame with columns theta, phi, omega_conformal, and V-columns for state

target_coords List with component z (complex number)

v_cols Character vector of V-column names

Value

Single-row data frame of the closest state (with angular_distance column added)

Examples

```
# Typically used with output from get_reachable_states
# find_closest_to_coords(states_df, target_coords, paste0("V", 1:n))
```

```
find_combination_in_states
```

Find a State in Reachable States Table

Description

Searches for a specific permutation state in a reachable states table and returns the first matching row with metadata.

Usage

```
find_combination_in_states(reachable_states_start, search_state)
```

Arguments

reachable_states_start Data frame with V-columns and metadata

search_state Integer vector, the state to search for

Value

Data frame row with state and metadata columns, or NULL if not found

Examples

```
df <- data.frame(V1 = c(1, 2), V2 = c(2, 1), operation = c("1", "2"),
                 step = c(1, 2), combo_number = c(1, 1))
find_combination_in_states(df, c(2, 1))
```

 find_path_bfs

Find Path via BFS Highways

Description

Builds BFS highway trees from start and final states, finds the closest pair of hub states (one from each highway), then uses find_path_iterative to connect them. Assembles the full path: bfs(start -> hub_s) + iterative(hub_s -> hub_f) + inverted_bfs(final -> hub_f)

Usage

```
find_path_bfs(
  start_state,
  final_state,
  k,
  bfs_levels = 500L,
  bfs_n_hubs = 7L,
  bfs_n_random = 3L,
  distance_method = "manhattan",
  verbose = TRUE,
  ...
)
```

Arguments

start_state	Integer vector, the starting permutation state
final_state	Integer vector, the target permutation state
k	Integer, parameter for reverse operations
bfs_levels	Integer, depth of sparse BFS from each side (default 500)
bfs_n_hubs	Integer, top-degree nodes per BFS level (default 7)
bfs_n_random	Integer, random nodes per BFS level (default 3)
distance_method	Character, "manhattan" or "breakpoints" (default "manhattan")
verbose	Logical, print progress (default TRUE)
...	Additional arguments passed to find_path_iterative

Value

List with path, found, cycles, bfs_info

find_path_iterative *Iterative Path Finder Between Permutation States*

Description

Finds a path between two permutation states using iterative cycle expansion. Generates random operation sequences, analyzes their cycles, and looks for intersections between forward (from start) and backward (from final) state sets. Uses bridge states to progressively narrow the search space.

Usage

```
find_path_iterative(
  start_state,
  final_state,
  k,
  moves = c("1", "2", "3"),
  combo_length = 20,
  n_samples = 200,
  n_top = 10,
  max_iterations = 10,
  potc = 1,
  ptr = 10,
  opd = FALSE,
  reuse_combos = FALSE,
  distance_method = "manhattan",
  sort_by = c("longest", "most_unique"),
  verbose = TRUE
)
```

Arguments

start_state	Integer vector, the starting permutation state
final_state	Integer vector, the target permutation state
k	Integer, parameter for reverse operations
moves	Character vector, allowed operations (default c("1", "2", "3"))
combo_length	Integer, length of random operation sequences (default 20)
n_samples	Integer, number of random sequences to test per iteration (default 200)
n_top	Integer, number of top sequences to analyze fully (default 10)
max_iterations	Integer, maximum number of search iterations (default 10)
potc	Numeric in (0,1], fraction of cycle states to keep (default 1)
ptr	Integer, max intersections to process per iteration (default 10)
opd	Logical, if TRUE filters states to only combos containing bridge state (default FALSE)

reuse_combos	Logical, if TRUE generates random combos only once per side (cycle 1) and reuses them in subsequent cycles. Saves time but reduces diversity (default FALSE)
distance_method	Character, method for comparing states during bridge selection. One of "manhattan" (sum of absolute differences) or "breakpoints" (number of adjacency violations). Default "manhattan".
sort_by	Character vector of sorting criteria for combo selection. See find_best_random_combinations for details. Default: c("longest", "most_unique").
verbose	Logical, if TRUE prints progress messages (default TRUE)

Details

Uses a compact C++ StateStore backend for O(1) incremental hash-indexed state storage, eliminating quadratic memory growth from repeated rbind.

Value

List containing:

path	Character vector of operations, or NULL if not found
found	Logical, whether a path was found
cycles	Number of iterations used
selected_info	Details about the selected intersection
bridge_states_start	List of forward bridge states
bridge_states_final	List of backward bridge states

Examples

```
# Small example
set.seed(42)
start <- 1:6
final <- c(3L, 1L, 2L, 6L, 4L, 5L)
# result <- find_path_iterative(start, final, k = 3, max_iterations = 5)
```

generate_state

Generate Reachable Random State

Description

Generates a random state reachable from 1:n by applying random operations (L, R, X). Guarantees the result is in the same connected component as the starting state.

Usage

```
generate_state(
  n,
  k = n,
  n_moves = 25L,
  moves = c("1", "2", "3"),
  max_attempts = 100L
)
```

Arguments

n	Integer, the size of the permutation
k	Integer, parameter for reverse_prefix operation
n_moves	Integer, number of random operations to apply (default 25)
moves	Character vector, allowed operations (default c("1", "2", "3"))
max_attempts	Integer, maximum attempts to generate a non-identity state (default 100)

Value

Integer vector representing a reachable permutation state

Examples

```
set.seed(42)
generate_state(10, k = 4)
generate_state(10, k = 4, n_moves = 100)
```

```
generate_unique_states_df
```

Generate Data Frame of Unique Random States

Description

Generates a data frame with unique random permutation states.

Usage

```
generate_unique_states_df(n, n_rows)
```

Arguments

n	Integer, size of each permutation state
n_rows	Integer, number of unique states to generate

Value

Data frame with n_rows rows and columns V1, V2, ..., Vn

Examples

```
set.seed(42)
df <- generate_unique_states_df(5, 10)
head(df)
```

get_reachable_states *Find Cycle in Permutation Group*

Description

Explores the Cayley graph starting from an initial state and applying a sequence of operations repeatedly until returning to the start state. Returns detailed information about all visited states, the cycle structure, and celestial LRX coordinates.

Usage

```
get_reachable_states(start_state, allowed_positions, k, verbose = FALSE)
```

Arguments

start_state	Integer vector, the initial permutation state
allowed_positions	Character vector, sequence of operations to repeat
k	Integer, parameter for reverse operations
verbose	Logical; if TRUE, prints progress and cycle information (default FALSE)

Value

List containing:

states	List of all visited states
reachable_states_df	Data frame with states, operations, steps, and celestial coordinates
operations	Vector of operations applied
coords	List of celestial coordinate objects per step
nL_total	Total left shifts
nR_total	Total right shifts
nX_total	Total reverse operations
total_moves	Total number of moves in the cycle
unique_states_count	Number of unique states visited
cycle_info	Summary string with cycle statistics

Examples

```
result <- get_reachable_states(1:10, c("1", "3"), k = 4)
writeLines(result$cycle_info)
```

`get_reachable_states_light`*Find Cycle Length (Lightweight Version)*

Description

Fast version of cycle detection that only returns cycle length and unique state count without storing all intermediate states. Useful for testing many operation sequences efficiently. Implemented in C++ for performance.

Usage

```
get_reachable_states_light(start_state, allowed_positions, k)
```

Arguments

<code>start_state</code>	Integer vector, the initial permutation state
<code>allowed_positions</code>	Character vector, sequence of operations to repeat
<code>k</code>	Integer, parameter for reverse operations

Value

List containing:

<code>total_moves</code>	Total number of moves to return to start state
<code>unique_states_count</code>	Number of unique states in the cycle

Examples

```
result <- get_reachable_states_light(1:10, c("1", "3"), k = 4)
cat("Cycle length:", result$total_moves, "\n")
cat("Unique states:", result$unique_states_count, "\n")
```

`invert_path`*Invert a Path of Operations*

Description

Reverses and inverts a sequence of operations. "1" (shift left) becomes "2" (shift right) and vice versa. "3" (reverse) stays the same.

Usage

```
invert_path(path)
```

Arguments

path Character vector of operations

Value

Character vector of inverted operations in reverse order

Examples

```
invert_path(c("1", "3", "2"))  
invert_path(c("1", "1", "3"))
```

manhattan_distance *Manhattan Distance Between Two States*

Description

Computes the sum of absolute differences between corresponding elements of two permutation states.

Usage

```
manhattan_distance(start_state, target_state)
```

Arguments

start_state Integer vector, first state
target_state Integer vector, second state

Value

Numeric, the Manhattan distance

Examples

```
manhattan_distance(1:5, 5:1)  
manhattan_distance(1:5, 1:5)
```

`manhattan_distance_matrix_gpu`*Compute Pairwise Manhattan Distance Matrix on GPU*

Description

Computes all pairwise Manhattan distances between two sets of states. Returns an $N1 \times N2$ matrix where entry (i,j) is the Manhattan distance between row i of `states1` and row j of `states2`.

Usage

```
manhattan_distance_matrix_gpu(states1, states2, batch_size = 256L)
```

Arguments

<code>states1</code>	Numeric matrix ($N1 \times n$), first set of states
<code>states2</code>	Numeric matrix ($N2 \times n$), second set of states
<code>batch_size</code>	Integer, number of <code>states2</code> rows to process at once (default 256)

Details

For large matrices, computation is batched over columns of the result to avoid GPU memory overflow.

Value

Numeric matrix ($N1 \times N2$) of Manhattan distances

Examples

```
if (cayley_gpu_available()) {  
  s1 <- matrix(c(1,2,3,4,5, 5,4,3,2,1), nrow = 2, byrow = TRUE)  
  s2 <- matrix(c(3,3,3,3,3, 1,1,1,1,1), nrow = 2, byrow = TRUE)  
  manhattan_distance_matrix_gpu(s1, s2)  
}
```

reconstruct_bfs_path *Reconstruct path from sparse BFS result*

Description

Traces back from target_key to the root (start state) using the parent_key/child_key edges in the BFS result.

Usage

```
reconstruct_bfs_path(bfs_result, target_key)
```

Arguments

bfs_result data.frame returned by sparse_bfs()
 target_key Character string — state key to trace back from

Value

Character vector of operations from start to target

reverse_prefix *Reverse First k Elements (with Coordinates)*

Description

Reverses the first k elements of the state vector (turnstile operation). Tracks celestial coordinates (LRX momentum).

Arguments

state Integer vector representing the current permutation state
 k Integer, number of elements to reverse from the beginning
 coords Optional list of current celestial coordinates. If NULL, starts from zero coordinates.

Value

List with components:

state Integer vector with first k elements reversed
 coords List of updated celestial coordinates (nL, nR, nX, theta, phi, omega_conformal)

Examples

```
result <- reverse_prefix(1:10, k = 4)
result$state
```

reverse_prefix_simple *Reverse First k Elements (Simple)*

Description

Simple prefix reversal without coordinate tracking.

Arguments

state Integer vector representing the current permutation state
k Integer, number of elements to reverse from the beginning

Value

Integer vector with first k elements reversed

Examples

```
reverse_prefix_simple(1:10, k = 4)
```

save_bridge_states *Save Bridge States to CSV*

Description

Writes a list of bridge states (each with state and cycle fields) to a CSV file.

Usage

```
save_bridge_states(bridge_states, filename)
```

Arguments

bridge_states List of lists, each containing state (integer vector) and cycle (integer)
filename Character, output CSV file path

Value

Invisible NULL. Side effect: writes a CSV file.

Examples

```
bs <- list(  
  list(state = 1:5, cycle = 0),  
  list(state = c(2, 1, 3, 4, 5), cycle = 1)  
)  
# save_bridge_states(bs, tempfile(fileext = ".csv"))
```

select_unique	<i>Select Unique States by V-columns</i>
---------------	--

Description

Removes duplicate rows based on state columns (V1, V2, ..., Vn).

Usage

```
select_unique(df)
```

Arguments

df	Data frame
----	------------

Value

Data frame with unique states

Examples

```
df <- data.frame(V1 = c(1, 1, 2), V2 = c(2, 2, 1), op = c("a", "b", "c"))
select_unique(df)
```

shift_left	<i>Shift State Left (with Coordinates)</i>
------------	--

Description

Performs a cyclic left shift on the state vector, moving the first element to the end. Tracks celestial coordinates (LRX momentum).

Arguments

state	Integer vector representing the current permutation state
coords	Optional list of current celestial coordinates. If NULL, starts from zero coordinates.

Value

List with components:

state	Integer vector with elements shifted left by one position
coords	List of updated celestial coordinates (nL, nR, nX, theta, phi, omega_conformal)

Examples

```

result <- shift_left(1:5)
result$state
result$coords

# Chain operations using coords
r1 <- shift_left(1:5)
r2 <- shift_left(r1$state, r1$coords)
r2$coords$nL

```

shift_left_simple	<i>Shift State Left (Simple)</i>
-------------------	----------------------------------

Description

Simple cyclic left shift without coordinate tracking.

Arguments

state Integer vector representing the current permutation state

Value

Integer vector with elements shifted left by one position

Examples

```
shift_left_simple(1:5)
```

shift_right	<i>Shift State Right (with Coordinates)</i>
-------------	---

Description

Performs a cyclic right shift on the state vector, moving the last element to the front. Tracks celestial coordinates (LRX momentum).

Arguments

state Integer vector representing the current permutation state

coords Optional list of current celestial coordinates. If NULL, starts from zero coordinates.

Value

List with components:

state	Integer vector with elements shifted right by one position
coords	List of updated celestial coordinates (nL, nR, nX, theta, phi, omega_conformal)

Examples

```
result <- shift_right(1:5)
result$state
```

shift_right_simple	<i>Shift State Right (Simple)</i>
--------------------	-----------------------------------

Description

Simple cyclic right shift without coordinate tracking.

Arguments

state	Integer vector representing the current permutation state
-------	---

Value

Integer vector with elements shifted right by one position

Examples

```
shift_right_simple(1:5)
```

short_path_bfs	<i>Shorten Path via Depth-Limited BFS Hopping</i>
----------------	---

Description

For each position along the path, explores all reachable states within depth BFS steps. If any of those states appear later in the original path (beyond current position + BFS steps taken), the algorithm "jumps" to the farthest such match, replacing the skipped segment with the shorter BFS route. States are indexed in a hash map supporting duplicate entries to catch the farthest possible jumps in paths with repeated states.

Usage

```
short_path_bfs(path, start_state, k, depth = 5L)
```

Arguments

path	Character vector of operations ("1"/"2"/"3" or "L"/"R"/"X")
start_state	Integer vector, the starting permutation state
k	Integer, parameter for reverse_prefix operation
depth	Integer, BFS exploration depth (default 5)

Value

List with path (shortened), original_length, new_length, savings

short_position	<i>Simplify Operation Path</i>
----------------	--------------------------------

Description

Removes redundant operations from a path: cancels inverse pairs ("1"+"2", "3"+"3"), reduces chains of shifts modulo n, and simplifies blocks between reverses.

Usage

```
short_position(allowed_positions, n)
```

Arguments

allowed_positions	Character vector of operations to simplify
n	Integer, size of the permutation ring (used for modular reduction)

Value

Character vector of simplified operations

Examples

```
short_position(c("1", "2"), n = 5)
short_position(c("3", "3"), n = 5)
short_position(c("1", "1", "1", "1", "1"), n = 5)
```

sparse_bfs	<i>Sparse BFS with Look-ahead and Hybrid Selection</i>
------------	--

Description

Sparse BFS with Look-ahead and Hybrid Selection

Usage

```
sparse_bfs(start_state, k, n_hubs = 7L, n_random = 3L, max_levels = 1000L)
```

Arguments

start_state	Integer vector — starting permutation
k	Integer — parameter for reverse_prefix operation
n_hubs	Number of top-degree candidates to keep per level (exploitation)
n_random	Number of random candidates to keep per level (exploration)
max_levels	Maximum BFS depth (default 1000)

Value

data.frame with columns: parent_key, child_key, operation, level

state_store_size	<i>Get State Store Size</i>
------------------	-----------------------------

Description

Returns the number of states currently stored in the StateStore.

Arguments

xp	External pointer to StateStore
----	--------------------------------

Value

Integer, number of stored states

Examples

```
store <- create_state_store(6L)
state_store_size(store)
```

store_add_from_df	<i>Add States to Store from Data Frame</i>
-------------------	--

Description

Converts a data.frame/data.table of states (as returned by analyze_top_combinations) into the compact C++ store.

Usage

```
store_add_from_df(store, df, cycle_val)
```

Arguments

store	External pointer to StateStore
df	Data frame with V1..Vn columns plus metadata
cycle_val	Integer, cycle number to assign

Value

Number of rows added (invisible)

store_analyze_combos	<i>Analyze Combinations Directly into Store</i>
----------------------	---

Description

C++ implementation that runs full cycle expansion for each combination and writes states + coordinates directly into the StateStore, bypassing data.frame creation entirely.

Usage

```
store_analyze_combos(store, top_combos, start_state, k, cycle_val)
```

Arguments

store	External pointer to StateStore
top_combos	Data frame with combination column
start_state	Integer vector, the initial permutation state
k	Integer, parameter for reverse operations
cycle_val	Integer, cycle number to assign

Value

Number of states added (invisible)

 store_analyze_combos_gpu

Analyze Combinations into Store Using GPU Batch Operations

Description

GPU-accelerated version of store_analyze_combos. Processes all combinations in parallel, step by step, grouping by operation type (L/R/X) for GPU matrix multiplication. Falls back to CPU if GPU is unavailable.

Usage

```
store_analyze_combos_gpu(store, top_combos, start_state, k, cycle_val)
```

Arguments

store	External pointer to StateStore
top_combos	Data frame with combination column
start_state	Integer vector, the initial permutation state
k	Integer, parameter for reverse operations
cycle_val	Integer, cycle number to assign

Value

Number of states added (invisible)

 store_clear_opd

Clear All OPD Filters

Description

Clear All OPD Filters

Usage

```
store_clear_opd(store)
```

Arguments

store	External pointer to StateStore
-------	--------------------------------

store_combos_for_state

Find Combo Numbers Containing a State in a Cycle

Description

Find Combo Numbers Containing a State in a Cycle

Usage

```
store_combos_for_state(store, state, target_cycle)
```

Arguments

store	External pointer to StateStore
state	Integer vector
target_cycle	Integer

Value

Integer vector of combo_numbers

store_filter_middle *Filter Middle States for a Cycle*

Description

Returns 0-based indices of states in the given cycle, excluding the first skip_first and last skip_last steps per combo.

Usage

```
store_filter_middle(store, target_cycle, skip_first = 5L, skip_last = 5L)
```

Arguments

store	External pointer to StateStore
target_cycle	Integer
skip_first	Integer (default 5)
skip_last	Integer (default 5)

Value

Integer vector of 0-based indices

store_find_best_match *Find Best Match by Manhattan Distance*

Description

Find Best Match by Manhattan Distance

Usage

```
store_find_best_match(store, target, candidate_indices = integer(0))
```

Arguments

store External pointer to StateStore
target Integer vector, target state
candidate_indices Integer vector of 0-based indices to search (empty = search all)

Value

Integer, 0-based index of best match

store_find_intersections
Find Intersections Between Two Stores

Description

Returns state keys present in both stores. $O(\min(N,M))$ via hash lookup.

Usage

```
store_find_intersections(store_a, store_b)
```

Arguments

store_a External pointer to StateStore
store_b External pointer to StateStore

Value

Character vector of common state keys

store_get_meta	<i>Get Metadata for a State</i>
----------------	---------------------------------

Description

Retrieves metadata (step, combo_number, cycle, operation, coordinates) for a single state by 0-based index.

Usage

```
store_get_meta(store, idx)
```

Arguments

store	External pointer to StateStore
idx	Integer, 0-based index

Value

Named list

store_get_state	<i>Get State from Store</i>
-----------------	-----------------------------

Description

Retrieves a single permutation state by 0-based index.

Usage

```
store_get_state(store, idx)
```

Arguments

store	External pointer to StateStore
idx	Integer, 0-based index

Value

Integer vector of length perm_length

store_lookup	<i>Lookup State Indices by State Vector</i>
--------------	---

Description

Lookup State Indices by State Vector

Usage

```
store_lookup(store, state)
```

Arguments

store	External pointer to StateStore
state	Integer vector

Value

Integer vector of 0-based indices

store_reconstruct_path	<i>Reconstruct Path from Store</i>
------------------------	------------------------------------

Description

Traces back through cycle chain using bridge states to build the correct operation sequence. Each bridge state defines which combo path to follow in each cycle.

Usage

```
store_reconstruct_path(
  store,
  bridge_states,
  target_state,
  target_cycle,
  target_combo
)
```

Arguments

store	External pointer to StateStore
bridge_states	List of bridge state entries, each with \$state (integer vector). Element 1 = root (cycle 0), element i+1 = bridge at cycle i.
target_state	Integer vector
target_cycle	Integer
target_combo	Integer

Value

Character vector of operations, or NULL

store_set_opd	<i>Set OPD Combo Filter for a Cycle</i>
---------------	---

Description

Restricts indices_for_cycle and filter_middle_indices to only return states from the specified combo_numbers for the given cycle.

Usage

```
store_set_opd(store, target_cycle, combos)
```

Arguments

store	External pointer to StateStore
target_cycle	Integer, cycle number to filter
combos	Integer vector of allowed combo_numbers

store_to_dataframe	<i>Convert Store to Data Frame</i>
--------------------	------------------------------------

Description

For debugging and backward compatibility. Converts the entire store contents to a data.frame with V1..Vn + metadata columns.

Usage

```
store_to_dataframe(store)
```

Arguments

store	External pointer to StateStore
-------	--------------------------------

Value

data.frame

`validate_and_simplify_path`*Validate and Simplify a Path*

Description

Verifies that a candidate path correctly transforms `start_state` into `final_state`, then attempts to simplify it. Returns the simplified path if it remains valid, otherwise the original.

Usage

```
validate_and_simplify_path(path_candidate, start_state, final_state, k)
```

Arguments

<code>path_candidate</code>	Character vector of operations
<code>start_state</code>	Integer vector, start state
<code>final_state</code>	Integer vector, target state
<code>k</code>	Integer, parameter for reverse operations

Value

List with components:

<code>valid</code>	Logical, whether the path is valid
<code>path</code>	Simplified or original path, or NULL if invalid

Examples

```
res <- validate_and_simplify_path(c("1", "3"), 1:5, c(5, 2, 3, 4, 1), k = 2)
res$valid
```

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